Three-Dimensional Scattering Of Internal Waves Off A Uniformly Sloping Bottom

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LONG-TERM GOAL

Any theory based on monochromatic linear wave dynamics is too simple and misleading to properly interpret or predict anything that involves a continuum of frequencies like a pulse of wave energy. Through hard case studies of scattering events of realistic non-random internal wavefields an important step will be taken towards developing an understanding of and capability to predict internal wavefield characteristics near sloping bottom topography and the generation of near-slope geostrophic currents due to the interaction of such wavefields with bottom topography.

OBJECTIVES

My objective is to determine what flows result in the vicinity of a uniformly sloping bottom when a three-dimensional realistic wave field encounters that slope. Present knowledge of such events are based on linear wave theory where single monochromatic waves are considered. In the case of constant buoyancy frequency *N* a single wave with frequency ω_i propagates at a fixed angle θ with the vertical with group velocity U_g and is either reflected forward or backward depending on whether $\omega_i < \omega_c$ or $\omega_i > \omega_c$ where ω_c is the critical frequency. At the critical frequency, where the angle θ associated with ω_i is equal to the angle of the slope, the group velocity of the outgoing wave goes to zero and wave-energy cannot propagate away. Amplitudes for the reflected wave become infinitely large and wavelengths go to zero. When a wave is reflected upslope, the propagation direction turns toward the upslope direction. These theories (Phillips 1963, 1977 and Eriksen 1982) consider steady state situations with wavefields that extent to infinity in all directions. There are no events with a ‘beginning' and an ‘end'. What occurs when a localized wavefield, both in space and time, reflects cannot be predicted using these simple theories. Neither are these theories sufficient to properly interpret Oceanic data. It is my objective to determine what can be expected to occur when such a wavefield impinges on a sloping bottom. I envision such a localized wavefield to be generated by strong meteorological forcing at the surface, a forcing of finite duration and horizontal scale.

APPROACH

Surface forcing is modeled by imposing a vertical velocity field at the surface which results from the divergence of a viscous Ekman layer generated by surface winds. Charney (1955) showed that this is the appropriate way to incorporate the wind-stress forcing for the case of a continuously stratified medium. The vertical Ekman pumping w_s at the base of the surface Ekman layer perturbs the underlying stratified Ocean and the perturbation propagates away as internal waves. Geostrophic currents can also result from this. The response is investigated with the linearized dynamics, the f-plane and Boussinesq approximation and a model stratification of constant N. First the Green's function G^s for the semi-infinite domain is determined. This gives the response to surface pumping infinitely concentrated at one position in space and time. The response of the fluid to arbitrary pumping w_s is then given by the convolution $G^s o w_s$ where `o' stands for the convolution integral over the surface and the time history of the surface pumping. The next step is to determine the Green's function G^n for the response to a given normal flow u_n at the slope. If we take for u_n the normal component of the velocity field $u=G^s o w_s$ at the slope, then $-G^n o w_n$ gives the reflected wave field. ‘o' stands for the convolution integral over the slope surface and the time history of the normal flow at the slope. The velocity field due to both the incoming and reflected wave field is then $u=G^s o w_s - G^n o w_n$. Once the Green's functions are calculated, the response to arbitrary forcing can be determined numerically whereas for special cases the response can sometimes be expressed in closed form.

WORK COMPLETED

The vector Green's function G^s for the semi-infinite domain has been calculated. It can be expressed in closed form in terms of Bessel functions. A variety of model forcings has been used and exact expressions for the asymptotic behavior (early time, large time, large distance from forcing region) have been found.

The vector Green's function G^n has to a large extent been determined but there are still some hard transforms that need to be calculated. For the normal velocity component and the pressure things are almost complete. For the along-slope and upslope velocity components some more calculations are to be performed. Already some interesting results regarding the near slope currents that are generated by a point source on the slope have been discovered.

RESULTS

I have analyzed in detail the early time behavior and the large time behavior of the Green's function G^s as well as the response of a semi-infinite Ocean to `switch on' point-sources and the response to finite-sized model surface. forcing like non-moving swirling atmospheric winds. The analysis has revealed a number of interesting features. Amongst others:

(a) in the case of surface pumping a radial outflow occurs near the surface which is initially irrotational as in a non-rotating unstratified fluid. The Coriolis force acts upon this flow which leads. to an acceleration in azimuthal direction. This leads to the growth of a vortical mode.

(b) the asymptotic structure of the vortical mode depends on the total mass flux at the surface but not on the time-history of the pumping

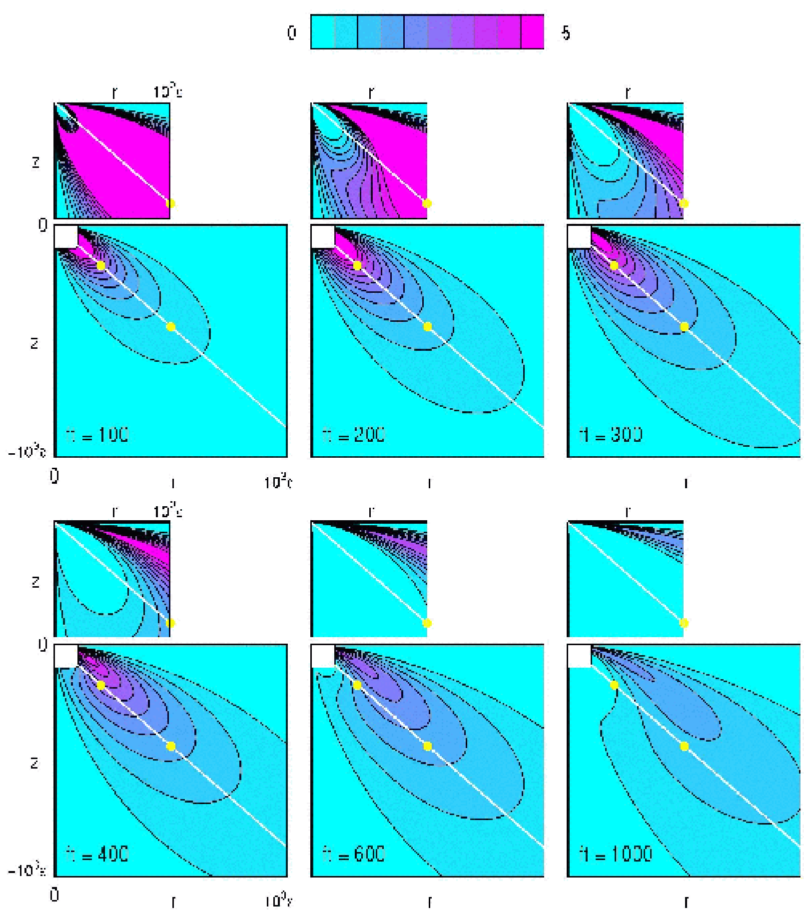
(c) for stronger stratification the amplitude of the vortical mode increases while the motion is more confined near the surface

(d) inertial oscillations and buoyancy oscillations are found to spread throughout the entire domain which shows that group velocity arguments can be fallacious

(e) the polarization relations for coherent linear wavefields do not hold

(f) for two very different initial conditions (unbalanced vortices) the same fraction of the initial total energy is converted into internal wave energy

(g) for forcing of finite spatial scale, at any position the internal gravity waves decay asymptotically faster than either the buoyancy or inertial oscillations. They propagate away from the forcing region as a distinct three-dimensional wavefield spreading throughout space. For forcing of finite duration the amplitudes at a fixed point in space evolve as $t^{1/2}\exp(-a(x,y,z)t)$ where (x,y,z) are the coordinates and a(...) is a positive function. In other words, the wavefield takes the form of a distinct ‘pulse'.



Wave energy density pulse due to an impulsive finite-sized forcing. Time is in inertial periods, rthe horizontal distance, z the depth. This is for N=2f. The scale of the forcing is epsilon.

IMPACT/APPLICATIONS

Numerical codes for simulating the response to surface forcing or the reflection of wave packets off a slope can be tested for accuracy and errors by contrasting the output with the exact results I derive. I have found that a number of paradigms based on group velocity arguments do not hold for wavefields with a continuum of wavevectors and frequencies. For example, the existence of inertial oscillations away from the surface is unexpected and near-surface sub-inertial oscillations occur underneath the forcing region. Also for finite-sized coherent forcings I find that the polarization relations for linear internal waves do not hold. In testing energy spectra in the internal wave frequency band for the existence of random ensembles of linear internal waves, the polarization relations are used to derive theoretical expressions that only depend on the frequency. Systematic deviations are found by Olbers' (1983). His observation that `this systematic disparity from internal wave kinematics points towards a complex contamination process' may not be correct. Since the Oceanic internal wave field is at least partially generated by coherent forcings like in this study, every data set will be `contaminated' by non-random internal waves, which when linear can violate the polarization relations. Eriksen (1982) when observing near-slope current ellipses `more narrow than would be expected from linear internal waves' may have fallen in the same trap. Coherent wavefields generated by surface-forcings of the kind I used have horizontal current ellipses more elongated at each frequency than a single plane monochromatic wave would have. Deviations from the theoretical predictions using the polarization relations may possibly be used to determine how much of the internal wavefield at a given location is truly random and how much is coherent. More generally this theoretical work will lead to a better understanding of the properties of non-random internal wave fields and currents near sloping topography.

RELATED PROJECTS

Dr. Carnevale of Scripps Institution of Oceanography, San Diego, is presently developing a finite-differences code to study the reflection of internal wave packets at a slope. This code can also be used to simulate the internal wave generation due to surface pumping at an upper rigid surface. The quasi-analytical solutions I have and will research will be used by Carnevale to assess the performance of his code, while I will benefit from a comparison with his fully nonlinear simulations, that is, it will show to what extent the linear results are valid.

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